
The marginal likelihood is a useful Bayesian model comparison criterion. For a complex model, the marginal likelihood is often analytically intractable. Chib (1995) exploits the fact that the marginal density can be expressed as the prior times the likelihood function over the posterior density. This simple identity holds for any parameter value. An estimate of the posterior density is shown to be available if all complete conditional densities used in the Gibbs sampler have closed-form expressions. Chib and Liazkov (2001) extend this method to overcome the problems associated with the presence of intractable full conditional densities. This extended method is developed in the context of MCMC chains produced by the Metropolis-Hastings algorithm, whose building blocks are used both for sampling and marginal likelihood estimation.

Liu et al. (2019) review Monte Carlo methods for computing the marginal likelihoods developed in the literature in recent years and provide a detailed development of how these methods are applied to the item response theory models. As empirically shown in Liu et al. (2019), the method of Chib (1995) is efficient but it requires a longer computational time. Inspired by the identity of Chib (1995), Zhang et al. (2017) develop a useful Conditional Predictive Ordinate (CPO) identity, which leads to the CPO decomposition to quantify the association between the longitudinal data and the survival data within the joint modeling framework.

**Other References**

