Abstract

The large sample solution to the problem of comparing the means of two (possibly heteroscedastic) populations, based on two random samples from the populations, hinges on the pivot underpinning the construction of the confidence interval and the test statistic being asymptotically standard Normal. We regularly use this well-known solution if the two samples are independent and the sample sizes are large. However, to establish the asymptotic standard Normality of the two sample pivot, existing results in the literature seem to assume, above and beyond the cross sample independence of the two samples, that the ratio of the sample sizes converges to a finite positive number. This the asymptotic behavior of the ratio of the sample sizes is impossible to verify in practical applications and carries the risk of rendering the theoretical justification of the large sample approximation invalid even in moderately unbalanced designs. Our results show that neither the restriction on the asymptotic behavior of the ratio of the sample sizes nor the assumption of cross sample independence is necessary for the asymptotic standard Normality of the two sample pivot. Convergence of the joint distribution of the standardized sample means to a spherically symmetric distribution on the plane, which has to be the bivariate standard Normal distribution, implies the asymptotic standard Normality of the two sample pivot, with the passage of the sample sizes to infinity being completely unrestricted. Finally, the two random samples we work with can be considered to be a truncation of an underlying infinite sequence of random vectors, with truncation in each coordinate occurring at a different stage. As long as this infinite sequence consists of independent (not necessarily identically distributed) elements, Cesàro convergence of the sequence of cross sample correlation coefficients to zero is equivalent to both the asymptotic standard Normality of the two sample pivot and the asymptotic bivariate standard Normality of the standardized sample means.