Poisson Approximation for Dependent Trials

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Let $A_1, A_2, ..., A_n$ be a sequence of dependent events. For $j = 1, 2, ...n$, define the indicator random variables

$$I_j = \begin{cases} 1, & \text{if the event } A_j \text{ occurs,} \\ 0, & \text{otherwise.} \end{cases}$$

In this article a new method is presented for approximating the distribution of

$$W = \sum_{j=1}^{n} I_j,$$

by a Poisson distribution and a derivation of a bound on the distance between the distribution of W and the Poisson distribution with mean $E(W)$. This new method is based on previous work by C. Stein where a central limit theorem for a sum of dependent random variables was obtained without the use of moment generating functions. In Chen (1975), the distance between the distribution of W and the Poisson distribution is assessed based the unique solution for an equation formulated specifically for the Poisson distribution. The focus of this work is on deriving upper and lower bounds on the distance between the distribution of W and the Poisson distribution with mean $E(W)$. The Poisson approximation itself has been used in several ares of probability and statistics including: DNA and protein sequences, geometrical probability, spatial statistics and reliability theory. Since the Poisson approximation can be inaccurate, improvements of the Poisson approximations have been investigated. Most importantly, a compound Poisson approximation method was developed that includes a bound on the distance between the distribution of W and the approximating compound Poisson approximation. The origin of this approach is included in the Chen (1975) article.